KAKATIYA INSTITUTE OF TECHNOLOGY & SCIENCE, WARANGAL

OFFICE OF THE PRINCIPAL

No. 008 /Acad/KITS/2021 Date: 06/08/2021

CIRCULAR

The Objective of Education is Learning, Not Teaching…
The Role of Teacher is to Facilitate Active Learning *(through effective teaching)*

Sub: POLICY DOCUMENT - Academic activities –Essential TLP –AY 2021-22– Reg.
Ref: All faculty meeting held on August 06, 2021

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Dear colleagues, Greetings!

The outcome of engineering education is to create industry-ready engineers.

Identify and list out the industry relevant knowledge, skills and qualities (KSQs) related to your course. Through effective course teaching-learning activities, you are expected to

(i) facilitate a successful active learning
(ii) make your course students ready for industry 4.0 imparting the relevant 21st century KSQs and
(iii) ensure that the identified KSQs are demonstrated by your course students through ***valid and reliable assessment & evaluation***

Also, in an attempt towards continuous improvement in our teaching-learning process (TLP), we are on an important mission to **integrate** innovation incubation research and entrepreneurship (**I²RE**) **into our course teaching to create industry-ready T-shaped engineers.**

In view of the above objectives, you are requested to focus on the following to facilitate active learning.

1. Course Introduction Video (CIV):

Why your students need to learn this course?

Prior to the commencement of class work, course faculty shall create a short 3–5minute course introduction video (CIV) and upload in **CourseWeb**. The objective of making CIV is to educate students on ***"Why do they need to learn your course?".***

In this CIV, the **course faculty shall cover** the following

(i) Welcome message with very short self-introduction
(ii) What KSQs will be imparted in learning your course
(iii) Course content and Course outcomes
(iv) Text book to be followed and Reference books for additional material
(v) CourseWeb page and its use for your course
(vi) ATLP – Additional assignments & solutions:
II/III/IV years: *Not for submission;* ***I years:*** *To submit for valuation.*
(vii) Assignments (A1, A2) to be submitted

(viii) Weekly one Tutorial class: *Problem solving sessions. Tutorial sheets will be uploaded in advance, students shall work on them prior to attending the class – INVITING possible alternative solutions from students during tutorial hour.*

(ix) Course Projects: *Prior to commencement of course, List of sample projects with abstract, required hardware/software, expected KSQs to be imparted and deliverables will be uploaded in CourseWeb. Interested student can work on one of these projects or* ***Students can come up with the ideas that interest them … Students should feel free to experiment with ideas that will help harness their enthusiasm***

(x) Special Assignments: why students should work on CRPs and CPs?

(xi) Self-learning topics: *Students are expected to take ownership of their learning. When students master certain basic content of course through self-learning, it allows teachers to facilitate active learning on technology, problem solving and critical thinking skills using higher order cognitive abilities. List of self learning topics shall be uploaded in the CourseWeb. As the course progress, the sources for self learning topic will be uploaded*

(xii) Academic Integrity and honesty ; Any other relevant information

In CIV, you need to tell, Why your students need to learn this course ?

2. Tutorial Classes:

Problem solving sessions using higher order cognitive abilities (HOCAs)

The Goal is to set your course students ***to move beyond simply recalling facts*** *(Remember, Understand) and* ***start using higher order cognitive abilities****(Apply, Analyze, Evaluate and Create)* ***for critical thinking and problem solving****.*

Week-wise tutorial sheets (one sheet /week) are to be prepared. Course faculty is expected to design most appropriate problems on intended cognitive domain learning level (CDLL) targeting the stated CO as per the Table of Specifications (ToS). Solutions are to be uploaded the following week.

(i) One class/week shall be devoted for tutorial classes

(ii) To be used for problem solving sessions using higher order cognitive abilities (HOCAs) – *Should not be converted to theory classes*

(iii) To complement student learning, the course faculty shall utilize these tutorial classes to enhance critical thinking and problem solving skills through higher order cognitive abilities (HOCAs) for students.

(iv) These tutorial classes shall be handled as one-to-one sessions

(v) Course faculty shall prepare **weekly tutorial sheets covering the previous week content of course** as per the **course worksheet.**
Week-1 tutorial will be Tutorial 1.

(vi) To impart the identified KSQs, the course faculty shall prepare tutorial sheets with well-designed problems on intended CDLL targeting the CO as per the ToS, covering the previous week content of the course, as per the worksheet.

(xiii) Course faculty shall upload the tutorial sheet, well in advance, in the CourseWeb and instruct students to come prepared to solve those problems in tutorial class of that week. This helps develop possible alternative solutions to the problems during tutorial hour.

(xiv) Instruct students to

a. maintain separate notebook for tutorials

b. write complete problem and solution*(so that students can comfortably refer to problems and solutions even when tutorial sheets are not with them)*

(vii) Course faculty to upload the detailed solutions in CourseWeb the following week or immediately after conduction of the tutorial class of that week.

(viii) Course faculty should check the student's tutorial notebook every week and affix signature with date

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

KAKATIYA INSTITUTE OF TECHNOLOGY & SCIENCE, WARANGAL

(An Autonomous Institute under Kakatiya University, Warangal)

Tutorial -1 (UNIT – I)

Course code - Name		**Branch- section**	
Tutorial Sheet posted in CourseWeb on	*Date-day*	**Tutorial class scheduled on**	*Date-day, Time*
Topics covered			

Tutorial	Problem No	Tutorial Problems	CO	CDLL
T-1	1.	**Problem on HOCAs - To be solved in the class** *Utilize these tutorial classes to enhance critical thinking and problem-solving skills through higher order cognitive abilities (HOCAs) for students.* *Engage students in solving the problem. See if they come up with possible alternative solutions*	CO 1	[Ap]
	2.	**A similar problem on HOCAs - to be given as tutorial homework problem to work on their own** *If capable, allow them to work on their own in the tutorial class itself*	CO 1	[Ap]
	3.	**Problem on HOCAs - To be solved in the class**	CO 1	[An]
	4.	**A similar problem on HOCAs - to be given as tutorial homework problem to work on their own**	CO 1	[An]
	5.	**Problem on HOCAs - To be solved in the class**	CO 1	[E]
	6.	**A similar problem on HOCAs - to be given as tutorial homework problem to work on their own**	CO 1	[E]

Tutorials are problem solving sessions using higher order cognitive abilities (HOCAs).
Prepare tutorial sheets with well-designed problems on intended CDLL targeting the CO as per the ToS, covering the previous week content of the course, as per the worksheet

3. Self-learning Topics:

Students are expected to take ownership and accountability of their learning

The course faculty shall identify self-learning topics for the total course and upload in the CourseWeb prior to the commencement of the course.

Students are expected to take ownership accountability of their learning. When students master certain basic content of course through self-learning, it allows teachers to facilitate active learning on technology, critical thinking and problem solving using higher order cognitive abilities (HOCAs), in class and tutorials

The objective is that the faculty has to effectively utilize the class teaching, facilitating activities which impart industry relevant knowledge and skills during the class, which is need of the hour.

With this objective in mind, the course faculty shall identify the list of self-learning topics for the total course and upload in the CourseWeb prior to the commencement of the course.

Students shall be informed that self-learning content is also a part of the course content and would be covered in assessment and evaluation.
Instruct students **to learn and master the content** by referring to the resources provided **before attending the connected class**.

List of Self-Learning Topics

S. No.	Unit	Self-Learning Topic

UPLOAD SELF LEARNING RESOURCES

As the course teaching progresses, the course faculty shall upload, in CourseWeb, the relevant learning resources on the identified self-learning topics. The resources shall include **posting pre-recorded video lectures explaining basics, explaining trivial derivations, explaining how to solve basic problems, self-explanatory hand-outs and other relevant material**.

Identify self-learning topics for the total course and upload in the CourseWeb prior to the commencement of the course.

As the course teaching progresses, upload in CourseWeb, the relevant learning resources.
Instruct students to learn and master the content by referring to the resources provided before attending the connected class.

Prepare students to take ownership and accountability of their basic learning. This is very important for life -long learning.

In the classes and tutorials, facilitate active learning on technology, critical thinking and problem solving using HOCAs.

4. ATLP – Additional Assignments:

Continuity in delivery of course content and student active engagement with course material

Assignment based Teaching Learning Process (ATLP) is very effective methodology to have continuity in the delivery of course content to the students and their active engagement with the course material. In short, the idea is

(i) **Assignment based teaching** by faculty and
(ii) **Assignment based learning** by students

For the students to leverage your class, you are **advised to follow the ATLP:**
Every class, start your lecture stating learning objectives.
Like… what students will learn in this class?

For example,
In this class, You will learn, what are the conditions necessary to apply Bernoulli's theorem? (LO1)
You will also learn, how to apply Bernoulli's theorem to solve the problems of flowing fluids? (LO2)
LO3: ….
LO4: ….

Outcomes of your lecture depend on learning objectives.
So, for every lecture fix the outcomes of lecture, matching the learning objectives. After completion of lecture, you are expected to check whether learning outcomes are covered.

Keeping this ATLP in mind, the course faculty is expected to Prepare ATLP-Weekly Assignment as per the class-wise format. The ATLP assignment is to be uploaded into Course Web on every SUNDAY, which shall contain questions from the content you planned to cover during forthcoming Monday to Saturday classes. ATLP assignments are to be prepared as per the format supplied and shall have

(i) questions testing class-wise outcomes.

(ii) questions of conceptual type and problems type to test the learning outcomes of that class(the pattern is as follows)

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ATLP – Assignment: Week-7

(UNIT – III)

Topics	: AC Circuits
Class	: B.Tech. I-Semester
Section	: STREAM-I (COMMON TO ALL BRANCHES)
Code-Subject	: **U18EE 105: Basic Electrical Engineering**
ATLP - Assignment posted on	: 24-01-2021
ATLP - Assignment Submission Due	**:** 31-01-2021

Class	Q.No	Answer the following questions, immediately after the class		CO	CDLL
Class-1	1.	Define the terms (i) Time Period and (ii) Frequency	[1]	CO 3	[R]
	2.	If v(t)=40Sin(20t+30) find amplitude, period and frequency	[2]	CO 3	[U]
Class-2	3.	Define the terms (i) Phasor (ii) Phase of a Phasor and (iii) Phasor difference	[1]	CO 3	[R]
	4.	Calculate the phase angle between $V_1(t)$= 10 Sin(ωt+45) and $V_2(t)$= 20 Sinωt and draw the phasor diagram.	[2]	CO 3	[U]
Class-3	5.	Convert the following into polar form (i) 10+j10 (ii) 0+j1 (iii) 1+j0 (iv) 10-j10	[1]	CO 3	[U]
	6.	If A=3+j4 and B=6+j8 find (i) A+B (ii) A-B (iii) A/B and (iv)A*B	[3]	CO 3	[U]

After completion of the class discussion, at the end, please refer to your ATLP assignment and questions of that class. You are advised to

1. State the class learning outcomes, like…*Having learned the conceptsof this lecture, you should now be able to*
   a. *….(related to LO1)*
   b. *….(related to LO2)*

2. Display/Refer to the ATLP Assignment of that week as last slide
3. Show your students those Qs covering your talk
4. Guide your students to solve those Qs on that day itself in a separate notebook in order to keep continuity in learning

Course faculty to post the detailed solutions to ATLP Weekly assignments in CourseWeb the following week.

At the end of each class, check with the students whether they can answer the ATLP assignment questions of today's class

ATLP ensures student engagement.

Course worksheet should emerge as a result of this ATLP approach.

5. Assignments A1, A2 (*to be submitted*):

Assignments should have problems which can be solved using HOCAs.

As a course faculty, your aim is to prepare students possessing industry relevant 21st century skills related to your course. Industry relevant KSQ requires HOCAs. Hence, **you are expected to design a valid and reliable assignment to enhance critical thinking and problem-solving skills using HOCAs**.
Try not to giving end-of-the-chapter questions from Text Book. Therefore, assignments shall consist of well-designed problems of appropriate CDLL targeting the CO as per the ToS, covering all the content of the specified units

Design assignments problems which can be solved using HOCAs.

With ATLP-additional assignments, assignments, tutorials and class room interactions, the faculty is expected to plan active learning activities and ensure that his course students acquire the KSQs required for both industry and higher education.

6. Real-world problems:

Real-world problems bring life to the course content
At least 5 real-world problems that can be solved using the course KSQs

As course faculty you need to prepare students for existing and emerging jobs and careers in the application areas of your course. You are expected to supplement the student learning by addressing the real-world problems.

Interact with industry experts. Identify,

(a) what are the industry relevant KSQs which are to be imparted related to the course?
(b) what are the real-world problems which can be solved completely or a part of the problem using the KSQs of your course?

Pick up at least 5 real-world problems. Expose students to those problems as part of course teaching, include them as part of tutorials, assignments, MSE and ESE questions. Questions shall start with good description of real-world problem. Then pose the question, which shall be a part of real-world problem, which can be solved using the course KSQs. Such problem-based learning engages students into active leaning.

Real-world problems bring life to the course content.
Course faculty is expected to supplement the student learning by addressing the real-world problems.
Discuss at least 5 real-world problems that can be solved using the course KSQs.

Create questions taking reference to those real-world problems and make them part of tutorial sheets, assignments, MSEs and ESE

7. Valid and Reliable Assessment & Evaluation:

Ensure your assessment reinforces teaching and learning

You have to ensure your students demonstrate the identified KSQs of the course.
How will you know -what KSQs your course students have achieved?
It will be known through a valid and reliable assessment.
The Table of Specification (ToS) of course helps ensure that **there is a match between what is taught and what is tested**. It helps the faculty to align learning objectives, instruction, assessment for outcomes. The ToS will serve as the blue print for assessment and the course faculty shall follow the ToS to improve the validity of assessment and evaluation.

The primary objective of assessment is to systematically test the cognitive levels of learning in a course. Therefore, unlike conventional questions, course faculty should design **quality questions to test intended CDLL targeting the stated CO, as per ToS, for tutorials, assignments, minor exams, mid semester exams and end-semester exams.**

Colleagues !,
When assessment is in alignment with the COs, the performance of students indicates the CO attainment, otherwise not !

Hence, as a course faculty, you have to build a better assessment system
(i) which will promote student learning
(ii) that measures the KSQ emphasized in the course

Test what has been taught.
Your question paper shows what you taught, *as per the KSQs.*
And quality of questions speaks about your quality of teaching.
Hence, quality assessment and accountable valuation are indicators of an effective teacher

8. Item Bank

Design item bank for your course…Test what has been taught !

Question paper (QP) is the basic tool used to measure educational achievement of a student. It is essential to **ensure quality of QP through certain attributes**, without which the QP becomes a poor instrument for measuring student achievement. **If the quality of tool (the QP) is poor, the assessment will be a poor indicator of outcomes**

An **item bank** is a repository of test questions (items). It is not just collection of test items (questions), but much more than that. **Item bank is a collection of quality test items designed according to the COs and the cognitive levels.**

Items are tagged with several parameters like CO, CDLL
The cognitive levels of assessment items shall never be compromised. Otherwise, it effects the quality of learning.

How many items to be designed ?
*A per the accreditation guidelines, the reasonable figure is **1:5***
At least 5 times the required number of questions are to be designed for ready inclusion in assessments.
You know, the course ToS specifies the CO and the CDLL for each of the items of the question paper.

For example:
In ESE, assume that you may have to set the following Qs targeting CO1
(a) One (1) question on Remember (R) level targeting CO1
(b) One (1) question on Understand (U) level targeting CO1
(c) Two (2) questions on Apply (Ap) level targeting CO1
(d) Two (2) questions on Analyze (An) level targeting CO1

As per 1:5, we need to design and keep the following number of items ready in the item bank matching the intended cognitive level targeting the specified CO
(a) Five (5) questions on Remember (R) level targeting CO1
(b) Five (5) questions on Understand (U) level targeting CO1
(c) Ten (10) questions on Apply (Ap) level targeting CO1
(d) Ten (10) questions on Analyze (An) level targeting CO1

These test items are to be designed at the time of your preparation for teaching the class. When you do this, you can focus your lecture on those intended cognitive levels.

While creating the test items, bear in mind that these items of a certain cognitive level, *for example 10 questions on Apply level targeting CO1*, should
(a) be of similar difficulty level
(b) require same time to solve them
(c) be good and of uniform quality

A test item is considered to be a **quality test item**, if it is designed appropriately at intended CDLL to test the attainment of targeted lecture outcome or CO.

It is worth mentioning here that during class interactions, to explain concepts, you will be citing several applications, analyze and evaluate information and talk about performance giving several typical examples. But we fail to give questions based on those skills in the exam, to test the students, and we end up asking just knowledge-based questions, which a student can answer with rote learning.

The reason is that most of the time, **we prepare QP on the spur of the moment, *without considering the quality of test items*. On the other hand, with hours of preparation, we go to class with very well-planned lecture to impart skills. This is where the mismatch is.**

A study indicated that most faculty were aiming their teaching and testing primarily at LOCAs. They are unaware of different cognitive levels and unable to understand the scope of HOCAs. The result is that they are unable to design quality test items.

In the instruction, driven by the assessment, we should have a match between what is taught and what is tested.Test items for assessing HOCAs shall be included more in number.

Faculty need to dedicate more time and effort to create questions at appropriate levels of cognition. Items on LOCAs take very little time to generate. It is almost impossible to generate test items on HOCAs, if the faculty write their test items the night before needed. Hence, these test items are to be designed at the time of your regular preparation for teaching the class.

Most times, we prepare QP on the spur of the moment, without considering the quality of test items.

As quality of question paper is direct indicator of what you taught and quality of your teaching, you are advised to create quality item bank for your course with well-designed test items (questions) of appropriate CDLL targeting the COs.

The test items of the bank shall be at a minimum of 1:5 ratio on the requirement.
These test items are to be designed at the time of your regular preparation for teaching the class, not the night before needed.

9. OBE Reports

The broad outcome of engineering education is to create industry-ready engineers

Ever since we became autonomous, we are meticulously working on outcome-based education (OBE) in our curriculum. The broad outcome of engineering education is to create industry-ready engineers.

The OBE is student centered learning method that focusses on student performance in terms of well-defined outcomes. The outcomes include knowledge, skills and qualities (KSQs).

The accreditation assures programme quality and the accrediting body - national board of accreditation (NBA) -follows outcome-based accreditation. The NBA defined the specifications of the KSQs of an engineer in terms of 12 programme outcomes (POs). All these POs

Expectation on students under OBE is that they should be able to apply the knowledge, analyze, evaluate the information and propose solutions to the real-world engineering problems.

At institute level, we have stated, in the programme educational objectives (PEOs) of our UG programmes, that within first few years of their graduation, our graduates would demonstrate technical expertise, soft skill, lifelong learning for a successful career.

Hence, as course faculty, fitting yourself into the OBE frame work, you need to

(a) identify the industry relevant 21st century KSQs which can be imparted with your course
(b) effectively integrate I^2RE into course teaching
(c) do effective teaching facilitating student learning
(d) expose course students to at least 5 real-world problems which can be solved, either completely or partly, using your course KSQs
(e) plan effective instruction driven by assessment
(f) create a quality item bank for your course
(g) design interactive quality assignments, tutorial sheets
(h) create quality question paper with items set at the relevant cognitive level targeting the stated CO
(i) closely look at the previous OBE reports of the course, talk to those faculty and plan your course teaching, assignments and tutorials
(j) allow continuous improvement in COAL, CDAL, CAL, POAL and curriculum

The broad outcome of engineering education is to impart 21st century KSQs and create industry-ready engineers.

As we progress in autonomy with accountable OBE, you should be able to design tailor made courses with content imparting the ever-changing industry relevant KSQs.

10. UG Project works / PG Dissertations

Impart the relevant KSQs to solve real-world complex engineering problem and prepare students to write a technical paper out of UG project work / PG dissertation

Major project offers an opportunity to integrate the knowledge acquired from various courses and apply it to solve real-world complex engineering problems.As the project supervisor, you are expected to

(a) Suggest well-defined project title
(b) define project objectives and expected deliverables
(c) help the students plan their project work and timeline
(d) provide enough resources for successful project completion
(e) impart the KSQs to propose solutions to the identified real-world problems

Dear colleagues,

To facilitate successful learning, for a 3 or 4 credit theory course, we prepare thoroughly beforehand and then facilitate student learning for somewhere around 40 to 50 hours of classes. In a 2 credit lab course, we do experiments on our own, take readings, verify results and then with lot of confidence, we facilitate around 24 to 26 hours of laboratory learning to students.

What should be our contribution in terms of preparedness and expertise for a 10 credit course ?

Major project credits = 3+7= It's a 10 credit course.

As project supervisor you have the responsibility of imparting the industry relevant 21st century KSQs. So, for a 10 credit major project course, how many hours of student learning is to be facilitated to complete the project ?
Keeping the quantum of efforts in mind, you are expected to create seriousness among the project students, emphasize on meeting cadence, project logbook, project timeline and guide the students to successful project execution and completion.

As one of the deliverables of project work / dissertation, you are advised to guide the students to successful project execution and completion, by imparting the relevant KSQs and make the students write a technical paper based on the solution(s) proposed, results obtained and prototype / working model / process /software package/system developed, for submission to a reputed non-predatory conference/non-paid peer reviewed journal (Scopus, SCI indexed journals)

You are advised to plan for minimum Scopus journal paper out of UG project and SCI journal paper out of PG dissertation. We can realize this, if we put efforts, as if we are working for our PhD degrees or post-doctoral research. In this process we work well beforehand, receive inputs from students, push out-of-the-box thinking, promote critical thinking, give clear guidelines to students, make them solve the problem using HOCAs and learn.

You are facilitator of learning in student-centered OBE.

In this 10-credit course, impart the KSQs to propose solutions to the identified real-world problems for UG project work / PG dissertation

Provide enough resources for successful project completion and convert the results of UG project / PG dissertation into non-paid Scopus/SCI journal papers.

Plan student learning activities and ensure that your team of students acquire the required KSQ to demonstrate the COs identified for UG Major project work / PG Dissertation

In the near future, we may have to earn increments and promotions based on the outcomes of our quality teaching and research.

11. Be a student's Teacher:

Keep motivation and engagement in assignments and tutorials
Bring relevancy to assignments and tutorials

A student's teacher creates a kind of engaged learning which leads to improved academic outcomes.

Try not to create a single - liner questions. Create questions introducingareal-world problem or an application and then pose the problem to be solved related to that application.

Many students may have difficulty in reading and understanding the content of the problem. Students should be exposed to reading skills in the context of examination. It poses a challenge because it requires students read and comprehend the text of the problem, identify the question that needs to be answered, and finally solve the problem.
Please find attached sample tutorial/assignment sheets posted by a professor for machine learning course at Purdue University. Look at the problems. Very interactive, motivating and engaging. Look at the lovely interactive introduction to each and every problem, still focusing on problem solving using HOCAs.

Dear colleagues,
Bring relevancy to assignments and tutorials.

Start every problem with a good motivating introduction giving reference to an application or real-world problem. Interactively fix what student has to do in that problem. Based on the complexity of the problem, give necessary hints, if it really requires for arriving at solution.

We must also follow such methodology in designing assignment problems, tutorial problems, MSE questions and ESE questions

Avoid giving single-liner questions.

For instance, analysis of our assignments will yield, a lot of times, disappointing results in terms of student motivation and engagement. But dear colleagues, w**e are capable of bringing relevancy to assignments,** when we

(a) *teach relevant quality content on identified KSQs*
(b) *use real-world problems and material to explore the topics*
(c) *describe the assignment/tutorial/exam questions depicting real -world problems or applications so that students need to read, understand and comprehend to solve the problem*
(d) *test what has been taught*

Dear colleagues,
Try coming out of that sphere of posting single-liner questions and also conventional end-of -the-chapter questions, which the students can answer with rote learning.

Our TLP should prepare students to solve real-world complex engineering problems using HOCAs. Start with a nice description for the problem depicting/referring to a real-world situation. Student should develop patience to read, understand and comprehend before start solving a problem. Create challenging problems which enhance critical thinking and problem-solving skills using HOCAs. Try not to give end-of-the-chapter questions from text book.

Be a student's teacher and bring relevancy to assignments / tutorials.

Avoid giving single-liner questions.

Start the problem with a good motivating introduction giving reference to an application or real-world problem. Interactively fix what student has to do in that problem.

In our efforts to bring relevancy to assignments and tutorials, the course faculty must think …

Does this assignment / tutorial interest students?
Whether I brought relevancy to assignments / tutorials?

An effective teacher facilitates active learning, fosters critical thinking, encourage creative work, enhance problem solving skills using HOCAs to impart the ever-changing industry relevant KSQs related to the course.

Prepare your students for existing and emerging careers in the application areas of your course.

Make tests more challenging by teaching and testing HOCAs

Let's create EDUCATION 4.0 for INDUSTRY 4.0
Let's make students' learning useful to solve real-world problems,
Not a Temporary-world of learning.
Let us add value to their lifelong learning for the ever-changing industry requirements.

Good Luck !
Happy Teaching!!

PRINCIPAL

Encls:
Sample tutorial/assignment sheets posted by a professor for machine learning course at Purdue University

To
All the faculty members through respective HoDs

Copy to:
1. *The Secretary & Correspondent*
2. *Dean, Academic Affairs*
3. *Coordinator IQAC*
4. *CoE*
5. *Dean R&D*
6. *Prof I/C OBE*
7. *AO*

Homework 0

Spring 2021
(Do not hand in)

The objective of this homework is to help you check if you have the necessary background for the course. The questions are about basic concepts in linear algebra, probability, optimization, and Python programming. For your reference, you can check the following textbooks:

- **Linear Algebra**: G. Strang, Linear Algebra and Its Applications, 4th Ed.
- **Probability**: S. Chan, Introduction to Probability for Data Science, 1st Ed. (This is my book. You can find it on my website.)
- **Optimization**: S. Boyd and L. Vandenberghe, Convex Optimization, 1st Ed.

Please complete this homework exercise. You do not need to submit the homework. We will have a 30-minutes dry run quiz between Jan-22 and Jan-23. The purpose of the quiz is to show you what to expect in our actual quiz. The dry run quiz will be based on this homework. Both this homework and the dry run quiz will not count towards your grade.

Exercise 1: Linear Algebra

One of the most common problems we ask in multivariate Gaussian is whether the model is valid. This leads to the question of whether the covariance matrix is positive definite or not. So what is positive definite? A real symmetric matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is said to be positive definite if you can verify at least one of the following conditions:

(i) $\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} > 0$ for all nonzero real nonzero vectors $\boldsymbol{x}$.

(ii) All the eigenvalues of $\boldsymbol{A}$ satisfy $\lambda_i(\boldsymbol{A}) > 0$, where $\lambda_i(\boldsymbol{A})$ denotes the i-th eigenvalue of $\boldsymbol{A}$.

(iii) All the upper left submatrices $\boldsymbol{A}_k$ have positive determinants, i.e., $|\boldsymbol{A}_k| > 0$, for all k.

(iv) All the pivots (without row exchange) satisfy $d_k > 0$. (Check Wikipedia on pivot.)

Now, consider the following matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. For what range of numbers a and b are the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ positive definite?

$$A = \begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{pmatrix}.$$

Exercise 2: Random Variable

In machine learning, it is quite common to handle one or more random variables. Thus, it is necessary to understand how to verify the distribution of some basic random variables. One very useful concept is the moment generating function. The moment generating function of a random variable X is defined as

$$M_X(s) = \mathbb{E}[e^{sX}], \tag{1}$$

where the expectation is taken over X. A few common moment generating functions can be found in my ECE 302 website, under the section Table. For example, if X is a Gaussian random variable with mean μ and variance σ^2, then the moment generating function of X is

$$M_X(s) = e^{\mu s + \frac{\sigma^2 s^2}{2}}.$$

Now, let X and Y be two independent Gaussian random variables. Assume X has mean μ_X and variance ${\sigma_X}^2$, Y has mean μ_Y and variance ${\sigma_Y}^2$. Define a new random variable $Z = X + Y$.

(a) Show that Z is a Gaussian random variable. Hint: Show that Z has a moment generating function having the same form of a Gaussian.

(b) Find the variance of Z.

Exercise 3: Probability

This is an exercise trying to link probability and linear algebra. Let $\boldsymbol{\Sigma} \in \mathbb{R}^{n \times n}$ be a symmetric positive definite matrix. We say that $\boldsymbol{\Sigma}$ has an eigen-decomposition if $\boldsymbol{\Sigma}$ can be written as

$$\boldsymbol{\Sigma} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T, \tag{2}$$

for some unitary matrices $\boldsymbol{U}$ such that $\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$, and diagonal matrix $\boldsymbol{\Lambda}$.

Consider a n-dimensional random vector $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$. Let $\boldsymbol{\Sigma} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T$ be the eigen-decomposition of $\boldsymbol{\Sigma}$. Let $\boldsymbol{Y} = \boldsymbol{U}^T\boldsymbol{X}$. Find the covariance matrix of $\boldsymbol{Y}$, defined below, in terms of $\boldsymbol{\Lambda}$.

$$\text{Cov}(\boldsymbol{Y}) \stackrel{\text{def}}{=} \mathbb{E}[\boldsymbol{Y}\boldsymbol{Y}^T] - \mathbb{E}[\boldsymbol{Y}]\mathbb{E}[\boldsymbol{Y}]^T.$$

Exercise 4: Optimization

Optimization plays a major role in this course. The most basic concept in constrained optimization is the notion of Lagrange multiplier. Consider minimizing a function

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{minimize}} \quad & f(\boldsymbol{x}) \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}. \end{aligned} \tag{3}$$

The Lagrangian of the problem is defined as

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \boldsymbol{\lambda}^T(\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}). \tag{4}$$

The solution of the original problem can be found by seeking a stationary point of the Lagrangian: $\nabla_{\boldsymbol{x}}\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}) = 0$, and $\nabla_{\boldsymbol{\lambda}}\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}) = 0$.

Solve the following optimization problem

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{minimize}} \quad & \|\boldsymbol{x} - \boldsymbol{y}\|^2, \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}. \end{aligned} \tag{5}$$

Express your answer in terms of $\boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{y}$.

Exercise 5: Python

We will be using Python for our course. To make sure that you at least know some basics about Python, we ask you to write a very simple program. If you do not have Python installed on your computer, you can try Google Colab.

In Python, import `numpy` and `matplotlib.pyplot`. Plot the function

$$f(x) = \frac{1}{1 + e^{-5(x-1)}}, \tag{6}$$

for $-5 \leq x \leq 5$. Make the linewidth of the curve 6 points. Use black color for the curve. Call x-label as `x`, y-label as `f(x)`, and title as `'my plot'`. Make sure you know how to download the plot and save it on your computer (mouse right click the figure, and click save image as).

Homework 1

Spring 2021
(Due: Friday, Feb 5, 2021, 11:59 pm Eastern Time)

Please submit your homework through **gradescope**. You can write, scan, type, etc. But for the convenience of grading, please merge everything into a **single PDF**.

Objective

To complete this homework assignment, you need to read the Background chapter of the lecture note, available on the course website under the page `note`. After this homework, you will be familiar with:

(a) Basic plotting tools in Python, for both 1D and 2D plots. Drawing random samples in Python.

(b) Cross-validation. See my book *Introduction to Probability for Data Science*, Chapter 3.2.5.

(c) Gaussian whitening. See my book Chapter 5.7.4.

(d) Basic ideas of regression. See my book Chapter 7.1.4.

You will be asked some of these questions in Quiz 1. The Quiz will be open on Feb 6, 8am Eastern Time, and close on Feb 7, 8am Eastern Time. The Quiz is 30 minutes long.

Exercise 1: Histogram and Cross-Validation

Let X be a random variable with $X \sim \mathcal{N}(\mu, \sigma^2)$. The PDF of X is written explicitly as

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{1}$$

(a) Let $\mu = 0$ and $\sigma = 1$ so that $X \sim \mathcal{N}(0, 1)$. Plot $f_X(x)$ using `matplotlib.pyplot.plot` for the range $x \in [-3, 3]$. Use `matplotlib.pyplot.savefig` to save your figure.

(b) Let us investigate the use of histograms in data visualization.

   (i) Use `numpy.random.normal` to draw 1000 random samples from $\mathcal{N}(0, 1)$.

   (ii) Make two histogram plots using `matplotlib.pyplot.hist`, with the number of bins m set to 4 and 1000.

   (iii) Use `scipy.stats.norm.fit` to estimate the mean and standard deviation of your data. Report the estimated values.

   (iv) Plot the fitted gaussian curve on top of the two histogram plots using `scipy.stats.norm.pdf`.

   (v) (Optional) Ask yourself the following questions: Are the two histograms representative of your data's distribution? How are they different in terms of data representation?

(c) A practical way to estimate the optimal bin width is to make use of what is called the **cross validation estimator of risk** of the dataset. Denoting $h = (\text{max data value} - \text{min data value})/m$ as the bin width, with m = the number of bins (assuming you applied no rescaling to your raw data), we seek h^* that minimizes the risk $\widehat{J}(h)$, expressed as follows:

$$\widehat{J}(h) = \frac{2}{h(n-1)} - \frac{n+1}{h(n-1)} \sum_{j=1}^{m} \widehat{p}_j^{\,2}, \tag{2}$$

where $\{\widehat{p}_j\}_{j=1}^m$ is the empirical probability of a sample falling into each bin, and n is the total number of samples.

(i) Plot $\widehat{J}(h)$ with respect to m the number of bins, for $m = 1, 2, ..., 200$.

(ii) Find the m^* that minimizes $\widehat{J}(h)$, plot the histogram of your data with that m^*.

(iii) Plot the Gaussian curve fitted to your data on top of your histogram.

Note: For additional discussions about this cross-validation technique, visit my book *Introduction to Probability for Data Science*, Chapter 3.2.5. More advanced materials can be found in the supplementary note of this homework.

Exercise 2: Gaussian Whitening

In this exercise, we consider the following question: suppose that we are given a random number generator that can only generate zero-mean unit variance Gaussians, i.e., $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$, how do we transform the distribution of $\boldsymbol{X}$ to an arbitrary Gaussian distribution? We will first derive a few equations, and then verify them with an empirical example, by drawing samples from the 2D Gaussian, applying the transform to the dataset, and checking if the transformed dataset really takes the form of the desired Gaussian.

(a) Let $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ be a 2D Gaussian. The PDF of $\boldsymbol{X}$ is given by

$$f_{\boldsymbol{X}}(\boldsymbol{x}) = \frac{1}{\sqrt{(2\pi)^2|\boldsymbol{\Sigma}|}} \exp\left\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right\}, \tag{3}$$

where in this exercise we assume

$$\boldsymbol{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{4}$$

(i) Simplify the expression $f_{\boldsymbol{X}}(\boldsymbol{x})$ for the particular choices of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ here. Show your derivation.

(ii) Using `matplotlib.pyplot.contour`, plot the contour of $f_{\boldsymbol{X}}(\boldsymbol{x})$ for the range $\boldsymbol{x} \in [-1, 5] \times [0, 10]$.

(b) Suppose $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$. We would like to derive a transformation that can map $\boldsymbol{X}$ to an arbitrary Gaussian.

(i) Let $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ be a d-dimensional random vector. Let $\boldsymbol{A} \in \mathbb{R}^{d\times d}$ and $\boldsymbol{b} \in \mathbb{R}^d$. Let $\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{b}$ be an affine transformation of $\boldsymbol{X}$. Let $\boldsymbol{\mu}_{\boldsymbol{Y}} \overset{\text{def}}{=} \mathbb{E}[\boldsymbol{Y}]$ be the mean vector and $\boldsymbol{\Sigma}_{\boldsymbol{Y}} \overset{\text{def}}{=} \mathbb{E}[(\boldsymbol{Y}-\boldsymbol{\mu}_{\boldsymbol{Y}})(\boldsymbol{Y}-\boldsymbol{\mu}_{\boldsymbol{Y}})^T]$ be the covariance matrix. Show that

$$\boldsymbol{\mu}_{\boldsymbol{Y}} = \boldsymbol{b}, \quad \text{and} \quad \boldsymbol{\Sigma}_{\boldsymbol{Y}} = \boldsymbol{A}\boldsymbol{A}^T. \tag{5}$$

(ii) Show that $\boldsymbol{\Sigma}_{\boldsymbol{Y}}$ is symmetric positive semi-definite.

(iii) Under what condition on $\boldsymbol{A}$ would $\boldsymbol{\Sigma}_{\boldsymbol{Y}}$ become a symmetric positive definite matrix?

(iv) Consider a random variable $\boldsymbol{Y} \sim \mathcal{N}(\boldsymbol{\mu}_{\boldsymbol{Y}}, \boldsymbol{\Sigma}_{\boldsymbol{Y}})$ such that

$$\boldsymbol{\mu}_{\boldsymbol{Y}} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\Sigma}_{\boldsymbol{Y}} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

Determine $\boldsymbol{A}$ and $\boldsymbol{b}$ which could satisfy Equation (5).

Hint: Consider eigen-decomposition of $\boldsymbol{\Sigma}_{\boldsymbol{Y}}$. You may compute the eigen-decomposition numerically.

(c) Now let us verify our results from part (b) with an empirical example.

(i) Use `numpy.random.multivariate_normal` to draw 5000 random samples from the 2D standard normal distribution, and make a scatter plot of the data point using `matplotlib.pyplot.scatter`.

(ii) Apply the affine transformation you derived in part (b)(iv) to the data points, and make a scatter plot of the transformed data points. Now check your answer by using the Python function `numpy.linalg.eig` to obtain the trasformation and making a new scatter plot of the transformed data points.

(iii) (Optional) Do your results from parts (c)(i) and (ii) support your theoretical findings from part (b)?

You can find more information about Gaussian whitening in my book *Introduction to Probability for Data Science*, Chapter 5.7.4.

Exercise 3: Linear Regression

Let us consider a polynomial fitting problem. We assume the following model:

$$y = \beta_0 + \beta_1 L_1(x) + \beta_2 L_2(x) + \ldots + \beta_p L_p(x) + \epsilon, \tag{6}$$

where $L_p(x)$ is the p-th Legendre polynomial, β_j are the coefficients, and ϵ is the error term. In Python, if you have specified a list of values of x, evaluating the Legendre polynomial is quite straight forward:

```
import numpy as np
from scipy.special import eval_legendre
x  = np.linspace(-1,1,50) # 50 points in the interval [-1,1]
L4 = eval_legendre(4,x)   # evaluate the 4th order Legendre polynomial for x
```

(a) Let $\beta_0 = -0.001$, $\beta_1 = 0.01$, $\beta_2 = 0.55$, $\beta_3 = 1.5$, $\beta_4 = 1.2$, and let $\epsilon \sim \text{Gaussian}(0, 0.2^2)$. Generate 50 points of y over the interval `x  = np.linspace(-1,1,50)`. That is,

```
x = np.linspace(-1,1,50) # 50 points in the interval [-1,1]
y = ... # fill this line
```

Scatter plot the data.

(b) Given the $N = 50$ data points, formulate the linear regression problem. Specifically, write down the expression

$$\widehat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\text{argmin}} \;\; \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|^2. \tag{7}$$

What are $\boldsymbol{y}$, $\boldsymbol{X}$, and $\boldsymbol{\beta}$? Derive the optimal solution for this simple regression problem. Express your answer in terms of $\boldsymbol{X}$ and $\boldsymbol{y}$.

(c) Write a Python code to compute the solution. Overlay your predicted curve with the scattered plot. For solving the regression problem, you can call `numpy.linalg.lstsq`.

(d) For the $\boldsymbol{y}$ you have generated, make 5 outlier points using the code below:

```
# ...
idx = [10,16,23,37,45]     # these are the locations of the outliers
y[idx] = 5                 # set the outliers to have a value 5
# ...
```

Run your code in (c) again. Comment on the difference.

(e) Consider the optimization

$$\widehat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\text{argmin}} \; \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|_1. \tag{8}$$

Convert the problem into a linear programming problem. Express your solution in the linear programming form:

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{minimize}} \;\; & \boldsymbol{c}^T\boldsymbol{x} \\ \text{subject to} \;\; & \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b}. \end{aligned} \tag{9}$$

What are $\boldsymbol{c}$, $\boldsymbol{x}$, $\boldsymbol{A}$, and $\boldsymbol{b}$?

(f) Solve the linear programming problem in Python using `scipy.optimize.linprog`, for the corrupted data in (d). Scatter plot the data, and overlay with your predicted curve. Hint: Remember to set `bounds=(None,None)` when you call `scipy.optimize.linprog`, because the variables in linprog are non-negative by default.

For this problem, you may want to check my book *Introduction to Probability for Data Science*, Chapter 7.1.4.

Exercise 4: Project: Check Point 1

By now I believe you should have read the course project instructions. If not, please read them now. The objective of this series of "check points" is to keep track of your progress, so that you will not wait until the last minute and then become panic. For check point 1, I want you to complete the following tasks:

1. **Latex**. I only accept final reports typed in Latex, using the ICML 2021 template. You can use overleaf (a free online platform) to type your report. Download the ICML template, put it in overleaf. Change the title to your project title, and change the author name to your name. (By default the ICML template uses the review mode. Please switch it to the camera ready mode.)

   When typing your name, please tell us: Your name, your major (e.g., ECE, AAE, etc), and your level (e.g., Online MS, PhD, Undergrad, etc)

2. **Topic**. There are four topics listed in the course website. Read them carefully. Think about which one you want to work on. Put a tentative title to your latex file. You title has to be informative about what *you* want to do. E.g., "Comparing Baseline Methods for Noise2Noise", "Re-Implementation of One-Size-Fits-All", etc.

3. **Datasets**. Every project will use some kind of datasets. Go to Google, search around, and find the most suitable datasets. List at least two possible datasets.

4. **Codes**. Some projects have existing codes on the internet. Find them out, and list them.

5. **References**. List at least one main references for your project, and at least 5 additional references. Follow the ICML instructions about the format of the .bib file.

Therefore, in this check point, I am expecting you to attach a one page PDF compiled from LaTeX (in ICML 2021 template). The one page should contain a title, your name, a one sentence summary of the project that you want to work on, a list of datasets you have found on the internet, the available codes you have found, and a list of references.

I understand that many of you have not yet decided which project to work on. You are free to change the project topic any time during the semester. You are also free to change the dataset, the code, the references, etc. As I said, the purpose here is to help you get started. Even if you have not yet decided, you should still do this exercise, because it will help you get familiar with the steps.

Homework 3

Spring 2021
(Due: Friday, Mar 5, 2021, 11:59 pm Eastern Time)

Please submit your homework through **gradescope**. You can write, scan, type, etc. But for the convenience of grading, please merge everything into a **single PDF**.

Objective

There are three things you will learn in this homework:

(a) Understanding why the maximum-likelihood estimate of the covariance matrix is the sample covariance.

(b) Implement a Bayesian decision rule for an image segmentation problem.

(c) Analyze the ROC curve.

You will be asked some of these questions in Quiz 3. The Quiz will be open on Mar 6, 8am Eastern Time, and close on Mar 7, 8am Eastern Time. The Quiz is 30 minutes long.

Exercise 1

Suppose that we are given a dataset $\mathcal{D} \stackrel{\text{def}}{=} \{\boldsymbol{x}_n\}_{n=1}^N$, where each sample $\boldsymbol{x}_n \in \mathbb{R}^d$ is an iid copy of the random variable $\boldsymbol{X}$. For simplicity, we assume that the distribution of $\boldsymbol{X}$ is a multi-dimensional Gaussian of mean $\boldsymbol{\mu} \in \mathbb{R}^d$ and covariance $\boldsymbol{\Sigma} \in \mathbb{R}^{d\times d}$. We further assume that the mean vector $\boldsymbol{\mu}$ is known and is given. Therefore, the likelihood of observing a sample $\boldsymbol{x}_n$ is fully controlled by the covariance matrix, i.e.,

$$p(\boldsymbol{x}_n \,|\, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\Big\{ -\frac{1}{2}(\boldsymbol{x}_n - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_n - \boldsymbol{\mu})\Big\} \tag{1}$$

Taking into consideration of all the samples in the dataset $\mathcal{D}$, the likelihood of $\mathcal{D}$ is

$$p(\mathcal{D} \,|\, \boldsymbol{\Sigma}) = \prod_{n=1}^N \left\{ \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\Big\{ -\frac{1}{2}(\boldsymbol{x}_n - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_n - \boldsymbol{\mu})\Big\} \right\}. \tag{2}$$

The goal of this analytical exercise is to derive the maximum-likelihood estimate of $\boldsymbol{\Sigma}$:

$$\widehat{\boldsymbol{\Sigma}}_{\text{ML}} = \underset{\boldsymbol{\Sigma}}{\text{argmax}} \quad p(\mathcal{D} \,|\, \boldsymbol{\Sigma}). \tag{3}$$

To make things simpler we assume that $\boldsymbol{\Sigma}$ and $\widetilde{\boldsymbol{\Sigma}} = \frac{1}{N}\sum_{n=1}^N (\boldsymbol{x}_n - \boldsymbol{\mu})(\boldsymbol{x}_n - \boldsymbol{\mu})^T$ are invertible in this exercise.

(a) Recall that the trace operator is defined as $\text{tr}[\boldsymbol{A}] = \sum_{i=1}^d [\boldsymbol{A}]_{i,i}$. Prove the matrix identity

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{x} = \text{tr}[\boldsymbol{A}\boldsymbol{x}\boldsymbol{x}^T], \tag{4}$$

where $\boldsymbol{A} \in \mathbb{R}^{d\times d}$.

(b) Show that the likelihood function in (3) can be written as:

$$p(\mathcal{D}|\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{Nd/2}} |\boldsymbol{\Sigma}^{-1}|^{N/2} \exp\left\{ -\frac{1}{2}\text{tr}\left[\boldsymbol{\Sigma}^{-1}\sum_{n=1}^N (\boldsymbol{x}_n - \boldsymbol{\mu})(\boldsymbol{x}_n - \boldsymbol{\mu})^T\right]\right\}. \tag{5}$$

(c) Let $\widetilde{\boldsymbol{\Sigma}} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{x}_n - \boldsymbol{\mu})(\boldsymbol{x}_n - \boldsymbol{\mu})^T$, and let $\boldsymbol{A} = \boldsymbol{\Sigma}^{-1}\widetilde{\boldsymbol{\Sigma}}$, and $\lambda_1, ..., \lambda_d$ be the eigenvalues of $\boldsymbol{A}$. Show that the result from the previous part leads to:

$$p(\mathcal{D}|\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{Nd/2}|\widetilde{\boldsymbol{\Sigma}}|^{N/2}}\left(\prod_{i=1}^{d}\lambda_i\right)^{N/2} \exp\left\{-\frac{N}{2}\sum_{i=1}^{d}\lambda_i\right\} \tag{6}$$

Hint: For matrix $\boldsymbol{A}$ with eigenvalues $\lambda_1, ..., \lambda_d$, $\text{tr}[\boldsymbol{A}] = \sum_{i=1}^{d}\lambda_i$.

(d) Find $\lambda_1, \ldots, \lambda_d$ such that (6) is maximized.

(e) With the choice of λ_i given in (d), prove that the ML estimate $\widehat{\boldsymbol{\Sigma}}_{\text{ML}}$ is

$$\widehat{\boldsymbol{\Sigma}}_{\text{ML}} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{x}_n - \boldsymbol{\mu})(\boldsymbol{x}_n - \boldsymbol{\mu})^T. \tag{7}$$

(f) What would be the alternative way of finding $\widehat{\boldsymbol{\Sigma}}_{\text{ML}}$? You do not need to prove. Just briefly mention the idea.

(g) If $\boldsymbol{\mu}$ is also estimated from the data so that it is $\widehat{\boldsymbol{\mu}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}_n$, the ML estimate $\widehat{\boldsymbol{\Sigma}}_{\text{ML}} = (1/N)\sum_{n=1}^{N}(\boldsymbol{x}_n - \widehat{\boldsymbol{\mu}})(\boldsymbol{x}_n - \widehat{\boldsymbol{\mu}})^T$ will a *biased* estimate of the covariance matrix because $\mathbb{E}[\widehat{\boldsymbol{\Sigma}}_{\text{ML}}] \neq \boldsymbol{\Sigma}$. Can you suggest an unbiased estimate $\widehat{\boldsymbol{\Sigma}}_{\text{unbias}}$ such that $\mathbb{E}[\widehat{\boldsymbol{\Sigma}}_{\text{unbias}}] = \boldsymbol{\Sigma}$? No need to prove. Just state the result.

Exercise 2

In this exercise I want you to implement a Bayesian decision rule for a (super classical) problem of image segmentation. The image we work with consists of a cat and some grass [1]. The size of this image is 500×375 pixels. The left hand side of Figure 1 shows the image, and the right hand side of Figure 1 shows a manually labeled “ground truth”. Your task is to do as much as you can to extract the cat from the grass, and compare your result with the ”ground truth”.

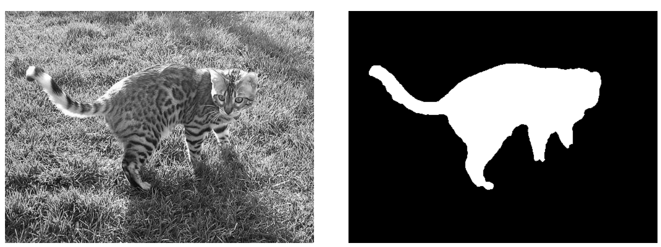

Figure 1: The “Cat and Grass” image.

Preparation Steps (No need to hand in)

First of all, go to the course website and download the data. Write the Python script to read the data and convert it into a data matrix.

```
train_cat   = np.matrix(np.loadtxt('train_cat.txt',   delimiter = ','))
train_grass = np.matrix(np.loadtxt('train_grass.txt', delimiter = ','))
```

The data matrices are $64 \times K_1$ and $64 \times K_0$, respectively, where K_1 is the number of training samples for Class 1 (cat), and K_0 is the number of training samples for Class 0 (grass).

Throughout this exercise, you need to read images and extract patches. To read an image, you can call `cv2` library or you can call `plt.imread`. For example, you can do

[1] Image Source: `http://www.robots.ox.ac.uk/vgg/data/pets/`

```
Y = plt.imread('cat_grass.jpg') / 255
```

The decision making of this problem is performed for every pixel. Therefore, you need to write a `for` loop to loop through all the pixels of the image. Moreover, you need to extract 8×8 neighbors surrounding each pixel. These can be done using the following commands:

```
M,N = Y.shape
for i in range(M-8):
    for j in range(N-8):
        block = Y[i:i+8, j:j+8] # This is a 8x8 block
        #
        # Something
        #
```

To make your life easier, it is okay to set the running index `i in range(M-8)` by neglecting the boundary pixels. In this case, the ground truth mask will have 8 rows and 8 columns less.

Your Tasks (Please hand in)

The Bayesian decision rule we are going to implement is based on the posterior distribution. We define the likelihood functions:

$$p_{\boldsymbol{X}|Y}(\boldsymbol{x}|C_1) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}_1|^{1/2}}\exp\Big\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_1)^T\boldsymbol{\Sigma}_1^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_1)\Big\},$$
$$p_{\boldsymbol{X}|Y}(\boldsymbol{x}|C_0) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}_0|^{1/2}}\exp\Big\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_0)^T\boldsymbol{\Sigma}_0^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_0)\Big\}, \tag{8}$$

and also the prior distributions $p_Y(C_1) = \pi_1$ and $p_Y(C_0) = \pi_0$. For simplicity, we assume that $\pi_1 = \frac{K_1}{K_1+K_0}$ and $\pi_0 = \frac{K_0}{K_1+K_0}$. The Bayesian decision rule says that

$$p_{Y|\boldsymbol{X}}(C_1|\boldsymbol{x}) \gtrless_{C_0}^{C_1} p_{Y|\boldsymbol{X}}(C_0|\boldsymbol{x}), \tag{9}$$

which is based on the **posterior** distribution.

(a) Substitute the multi-dimensional Gaussian likelihood (8) and the priors π_1 and π_0 into (9). Show that the decision rule is equivalent to

$$-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_1)^T\boldsymbol{\Sigma}_1^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_1) + \log\pi_1 - \frac{1}{2}\log|\boldsymbol{\Sigma}_1| \gtrless_{C_0}^{C_1} -\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_0)^T\boldsymbol{\Sigma}_0^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_0) + \log\pi_0 - \frac{1}{2}\log|\boldsymbol{\Sigma}_0|.$$

(b) Estimate $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_0$, $\boldsymbol{\Sigma}_1$, $\boldsymbol{\Sigma}_0$, π_1 and π_0 in Python. Report:

(i) The first 2 entries of the vector $\boldsymbol{\mu}_1$ and the first 2 entries of the vector $\boldsymbol{\mu}_0$.

(ii) The first 2×2 entries of the matrix $\boldsymbol{\Sigma}_1$ and the first 2×2 entries of the matrix $\boldsymbol{\Sigma}_0$.

(iii) The values of π_1 and π_0.

(c) Write a double for loop to loop through the pixels of the testing image. At each pixel location, consider a 8×8 neighborhood. This will be the testing vector $\boldsymbol{x} \in \mathbb{R}^{64}$. Dump this testing vector $\boldsymbol{x}$ into the decision rule you proved in (a), and determine whether the testing vector belongs to Class 1 or Class 0. Repeat this for other pixel locations.

```
for i in range(M-8):
    for j in range(N-8):
        block = Y[i:i+8, j:j+8]
        #
```



```
        # Something
        #
        prediction[i,j] = # Something
```

If you do everything right, you will get a binary image. Submit this predicted binary image. Remark: My program runs for about 10-15 seconds. If your code takes forever to run, something is wrong.

(d) Consider the ground truth image `truth.png`. Report the mean absolute error (MAE) between your prediction and the ground truth:

$$\text{MAE} = \frac{1}{\#\text{ of pixels}} \sum_{i,j} \left| \texttt{prediction}[i,j] - \texttt{truth}[i,j] \right|.$$

Report your MAE. Remark: Because we are not dealing with the boundary pixels (which explains why I set `i in range(M-8)`), when computing the MAE you need to set the true mask to `truth[0:M-8, 0:N-8]`.

(e) Go to the internet and download an image with similar content: an animal on grass or something like that. Apply your classifier to the image, and submit your resulting mask. You probably do not have the ground truth mask, so please just show the predicted mask. Does it perform well? If not, what could go wrong? Write one to two bullet points to explain your findings. Please be brief.

Exercise 3

The objective of this exercise is to plot the ROC curve. You may want to read Chapter 9.4 and Chapter 9.5 of my book.

(a) The Bayesian decision rule you derived in Exercise 2 is actually equivalent to the likelihood ratio test:

$$\frac{p_{\boldsymbol{X}|Y}(\boldsymbol{x}|C_1)}{p_{\boldsymbol{X}|Y}(\boldsymbol{x}|C_0)} \gtrless_{C_0}^{C_1} \tau, \tag{10}$$

for some threshold constant τ. Determine τ that corresponds to the decision rule in Exercise 2.

(b) Implement this likelihood ratio test rule for different values of τ. For every τ, compute the number of true positives and the number of false positives. Then, we can define the probability of detection $p_D(\tau)$ and the probability of miss $p_F(\tau)$ as:

$$p_D(\tau) = \frac{\#\text{ true positives}}{\text{total }\#\text{ of positives in ground truth}}$$
$$p_F(\tau) = \frac{\#\text{ false positives}}{\text{total }\#\text{ of negatives in ground truth}}.$$

Plot the ROC curve. That is, plot $p_D(\tau)$ as a function of $p_F(\tau)$. Your ROC curve should cover the range $[0,1] \times [0,1]$. Remark: Generating this ROC curve will take a minute or two.

(c) On your ROC curve, mark a red dot to indicate the operating point of the Bayesian decision rule.

(d) Implement a linear regression classifier for this problem, and plot the ROC curve. The idea is to construct a matrix system:

$$\underbrace{\begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_0 \end{bmatrix}}_{=\boldsymbol{A}} \boldsymbol{\theta} = \underbrace{\begin{bmatrix} \mathbf{1} \\ -\mathbf{1} \end{bmatrix}}_{=\boldsymbol{b}},$$

where $\boldsymbol{X}_1 \in \mathbb{R}^{K_1 \times d}$ and $\boldsymbol{X}_0 \in \mathbb{R}^{K_0 \times d}$ are the training data matrix of Class 1 and Class 0. Solve the regression problem

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \mathbb{R}^d}{\text{argmin}} \, \|\boldsymbol{A}\boldsymbol{\theta} - \boldsymbol{b}\|^2.$$

During testing, write a double for loop to through all the 8×8 neighbors of the image pixels. The decision rule per neighbor is

$$\widehat{\boldsymbol{\theta}}^T \boldsymbol{x} \quad \gtrless_{C_0}^{C_1} \quad \tau, \tag{11}$$

where $\widehat{\boldsymbol{\theta}}$ is the trained model parameter, and $\boldsymbol{x}$ is the testing 8×8 neighbor. By varying the threshold τ, you can obtain another ROC curve. Plot it.

Exercise 4: Project Check Point

This is check point # 3. As we go to this check point, you should have become familiar with your project. You should have known the basic literature, and you should have played some existing codes. The goal of this check point is to jump into the core problem by making some small modifications to the existing code. Because everyone has a slightly different project goal, I can only offer some very generic advices:

- If your proposed theme requires training on a different dataset, it is the time to program it and try training your model using the new dataset (and new configurations).
- If you want to compare several baseline methods, it is the time to setup an infrastructure and define the evaluation benchmark. Make sure that the comparisons you are making are fair to all competing methods.
- If you want to attack classifier, you should have an attacker ready. If you want to defend a classifier, you need to start thinking about how to do adversarial training.
- If you want to re-implement a label noise algorithm, start with the simplest toy problem (e.g., MNIST or even smaller dataset) and consider the simplest model (few layers of networks). Implement something.

You will certainly hit a lot of obstacles because you are still learning the code. This is normal, and you need to overcome it. No matter what you do, please keep a track record of your **quantitative experiments**. What experiments have you done? How do you know your training is on the right track? Do you have any training loss curve? Do you have any baseline benchmark that is working? Any troubleshoot plan?

Please focus on one specific (small) task and do not be too ambitious. I am not asking you to win the Turing award. I am asking you to learn something.

Append a one-page summary of the things you have done in check point #3 to the previous check point # 1 and #2. (So altogether you should have 3 pages after check point #3.) They do not need to contain any complete story, but you need to put down the plots, tables, figures, etc. Write a few major bullet points.